PERGAMON

International Journal of Heat and Mass Transfer 45 (2002) 3681–3688

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

An experimental and theoretical investigation into the hyperbolic nature of axial dispersion in packed beds

V. Sastha Prasad, S.K. Das *

Heat Transfer and Thermal Power Laboratory, Department of Mechanical Engineering, Indian Institute of Technology, Madras, Chennai 36, India

Received 14 April 1998; received in revised form 28 February 2002

Abstract

The concept of hyperbolic axial dispersion of heat in a flowing fluid which is known as 'third sound wave' has been examined taking packed bed as an example. This technique analyse fluid flow and heat transfer by introducing an axial dispersion term in the one-dimensional energy equation to take care of flow maldistribution and backmixing. The present approach models this dispersion in terms of two parameters which are proposed to follow hyperbolic conduction law. A regenerator bed consisting of stainless steel wire mesh packing has been used to carry out experiments for the purpose of validating the concept. The analytical model presented uses a Laplace transform technique for the solution of simplified energy equation. The computed outlet fluid temperature is compared with experimental output and the two model parameters, dispersive Peclet number (*Pe*) and its propagation velocity (c^*) are estimated. The present model, the parabolic (Fourier) dispersion model and the non-dispersive plug flow model are compared with the experimental result which brings out the closeness of the present proposition to reality compared to other models. A standard experimental technique is suggested which can be used for the measurement of parameters related to axial dispersion. The present study is the first experimental evidence of 'third sound wave' in fluids and lays foundation of this proposition.

1. Introduction

Packed bed regenerators are one of the important heat transfer equipment which are used in high temperature applications such as gas turbine power plants as well as in low temperature applications such as cryogenic liquefaction systems. The concept of axial dispersion in first place as proposed by Taylor [1] and its further development by Roetzel and Spang [2] in the form of 'hyperbolic dispersion' subsequently, were aimed at the removal of a major deficiency of the contemporary studies in which the fluid was assumed to have 'plug flow' and the effects of flow maldistribution and backmixing were not considered. Adebiyi and Chenevert [3] reviewed all such one-dimensional models applied to a packed bed thermal energy storage system. Their review reaffirms the fact that beginning with the pioneering work of Schumann [4] all the subsequent investigators including Riaz [5], Monta Khab [6] or Lu [7] have paid little attention towards the effects of flow maldistribution and backmixing. Since the flow maldistribution and backmixing bring about significant deviation from one-dimensional velocity and temperature profile, accurate simulation of the temperature response is not possible by using plug flow model which is usually used for the design and control of a large family of heat transfer equipment. Usually flow maldistribution is less severe in packed beds compared to open type equipment. Hence, any successful model for the analysis of such systems will be even more attractive for other compact equipment where maldistribution is acute.

The concept of dispersion was introduced to analyse the transient behaviour of multipass shell and tube heat exchangers by Roetzel and Xuan [8]. The dispersion phenomenon was proposed to follow Fourier law of conduction in the one-dimensional energy equation.

* Corresponding author. Tel.: +91-44-4458550.
E-mail address: sarit_das@hotmail.com (S.K. Das).

0017-9310/02/$ - see front matter
PII: S0017-9310(02)00099-6

Nomenclature

A	convective heat transfer area (m^2)
A_f	fluid free flow cross sectional area (m^2)
A_m	cross-sectional area of the solid matrix (m^2)
$\overline{A}$	4×4 coefficient matrix
c	conduction wave propagation velocity (m/s)
c^*	dispersion wave propagation velocity (m/s)
C_m	specific heat of the solid (J/kg K)
C_p	isobaric specific heat of the fluid (J/kg K)
G	exchanger flow stream mass velocity (kg/m^2 s)
h	heat transfer coefficient (W/m^2 K)
k	thermal conductivity of the solid matrix (W/m K)
L	length of the bed (m)
m_f	mass flow rate of fluid (kg/s)
m	mass of hold up fluid over the bed at a particular instant (kg)
M	mass of solid matrix (kg)
NTU	number of transfer units $= hA/m_f C_p$
p	porosity of the wire screen matrix
Pe	dispersive Peclet number $= m_f C_p L/\lambda A_f$
P_m	matrix Peclet number $= m_f C_p L/k A_f$
q	heat transfer by conduction in the axial direction (W/m^2)
q_x	heat transfer by dispersion in the axial direction (W/m^2)
Re	Reynolds number $= 4 r_h G/\mu$
r_h	hydraulic radius (m)
s	transformed time variable in Laplace domain
T	temperature (K)
u	velocity of the fluid (m/s)
u_{ij}	eigenvectors of $\overline{A}$ matrix
V	third sound Mach number $= u/c^*$
x	space co-ordinate (m)
X	dimensionless space co-ordinate $= x/L$
z	non-dimensional time $= m_f C_p t/M C_m$
Ω	the dimensionless time at which inlet fluid temperature reaches the maximum, $m_f C_p t_{max}/M C_m$
α	thermal diffusivity (m^2/s)
α^*	thermal diffusivity of axial dispersion $= \lambda/\rho C_p$ (m^2/s)
β_j	eigenvalues of $\overline{A}$ matrix
θ	non-dimensional temperature $= (T - T_{atm})/(T_{in,max} - T_{aim})$
λ	axial dispersion coefficient of fluid (W/m K)
ρ	density of the fluid (kg/m^3)
τ	time (s)
τ_{max}	time required to reach maximum temperature at inlet (s)
μ	viscosity (N s/m^2)
η	heat capacity ratio $= M C_m/m C_p$

Subscripts

atm	at atmosphere
f	of the fluid
in	at the inlet
in, max	at inlet, maximum
m	solid matrix
+	the section just after the heat exchanger inlet
−	the section just before the heat exchanger inlet

Xuan and Roetzel [9] developed a versatile and efficient method to predict the dynamic performance of parallel and counter flow heat exchangers subject to arbitrary temperature variations and step flow variations, considering dispersion to include the effect of flow maldistribution. Further, Roetzel et al. [10] developed an oscillating temperature technique to measure the heat transfer coefficient and axial dispersion coefficient. Das et al. [11] carried out experiments to predict the transient response of plate heat exchangers subject to a step change in the temperature from a uniform state considering dispersion in the fluid. Roetzel and Balzereit [12] introduced residence time measurements for the determination of axial dispersion coefficients in plate heat exchangers.

The concept of hyperbolic dispersion was introduced to rectify the problem of under prediction of time delay of the conventional (parabolic) dispersion model. This was taken care by introducing a finite propagation velocity in the dispersion equation (Fourier law of apparent conduction) in the form

$$q_1 + \frac{\alpha^*}{c^{*2}} \frac{Dq_x}{D\tau} = -\lambda \frac{\partial T_r}{\partial x} \tag{1}$$

where the substantative operator appearing due to fluid movement is given by

$$\frac{D}{D\tau} = \frac{\partial}{\partial \tau} + u \frac{\partial}{\partial x} \tag{2}$$

In Eq. (1) λ is the dispersion coefficient, α^* is the apparent thermal diffusivity of due to dispersion in fluid and c^* is the dispersion wave propagation velocity.

The only evidence of hyperbolic dispersion concept so far has been the indirect comparison with numerical simulation of laminar flow through a tube by Roetzel et al. [13]. They named this mode of dispersion as 'third sound wave'. This is because while the normal acoustic wave can be called 'first sound wave', it is customary to call thermal wave in the form of non-Fourier (or hyperbolic) conduction as the 'second sound wave'. Thus by analogy with non-Fourier conduction, the hyperbolic dispersion can be termed as also a sound wave, the 'third' one.

In the present work, the concept of hyperbolic dispersion is validated experimentally by applying it to the transient analysis of a packed bed. The axial dispersion coefficient and its propagation velocity are measured for three different sizes of wire mesh. The present work also suggests a standard experimental technique for the measurement of parameters related to hyperbolic axial dispersion.

2. Experimental set up and procedure

Fig. 1 shows the schematic diagram of the experimental set-up. The experimental set up consists of a test section of 60 mm length and 100 mm diameter galvanised iron tube with flanges in which stainless steel wire mesh (16×16 or 10×10 or 5×5) is packed over a length of 40 mm, centrally. This flanged section is a part of a long pipe line from the blower to have fully developed flow at the flow measurement point. It may be mentioned here that the bed aspect ratio has been kept low to conform to typical regenerator bed where common blower is used instead of expensive compressor. However, during experiment it was observed that change of bed depth from 30 to 60 mm does not appreciably change the dispersion parameters evaluated. The temperature of fluid at inlet to the bed, outlet to the bed and three different cross sections inside the matrix are measured using K type (Chromel–Alumel) thermocouples. Air from a blower of 15 HP capacity is passed through a 100 mm diameter tube. Air is first heated using a 2 kW electric heater which is wrapped outside the tube. A burner is used as an additional source of heat. The burner tip is kept inside a 150 mm diameter and 450 mm long shroud which is closed at one end. The flue gases are allowed to pass over the tube through which air is flowing making it a flue gas to test fluid (air) heat exchanger (11). Temperature of air after the heating zone is measured using a thermocouple. Following the thermocouple a bypass valve is provided and the air is bypassed till its temperature reaches steady state. After the bypass valve, a butterfly valve is provided in the main line which is followed by the test section. Using the butterfly valve heated air is prevented from entering the test section till it reaches a steady temperature. The flow rate of air is measured using a calibrated orifice meter. The range of flow rates used was 100–250 m^3/h. Thermocouples are connected to data acquisition system which contains a data acquisition A/D card with counter and cold junction compensation along with appropriate software. The test section is cooled using compressed air till the butterfly valve is opened to maintain a uniform initial temperature which heats up because of heat leak.

The measured values of temperatures, time and axial distance are non-dimensionalised using the non-dimenianalised parameters of temperature, time, distance and solid matrix Peclet number (P_m).

A polynomial of the form $\theta_{in} = (a_0 + a_1 z + a_2 z^2 + a_3 z^3)[1 - H(z - \Omega)] + H(z - \Omega)$ (where Ω is the dimensionless time at which inlet fluid temperature reaches the maximum and $H(z - \Omega)$ is the unit step function) is fitted to the recorded inlet temperature history which is shown in Fig. 2 and in the subsequent computation, this real input function instead of an idealised step function has been used.

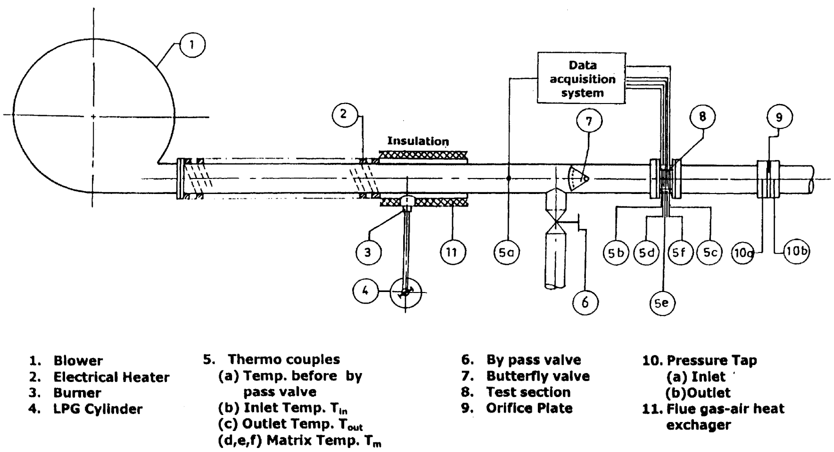

Fig. 1. Schematic diagram of the experimental set-up.

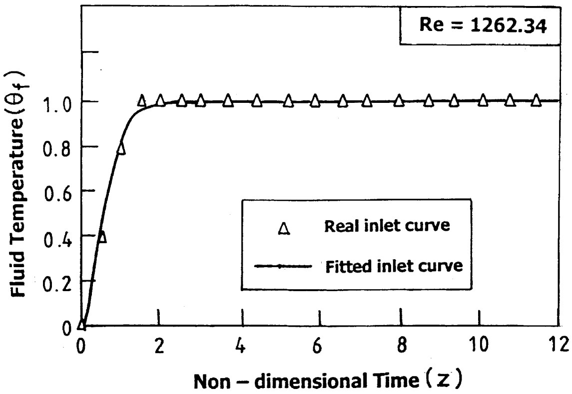

Fig. 2. A sample of measured inlet fluid temperature and fitted curve.

In measurement, the main source of error has been the thermocouple response delay, its accuracy and the error in flow measurement by the orifice meter. It was found that the data acquisition unit can handle thermocouple response with an accuracy of 0.1 °C which is about 0.7% of the temperature difference achieved. The time constant is 50 ms which is much less than the measurement time. The maximum uncertainty in temperature measurement is found to be ±3%. It is estimated that the maximum uncertainty in flow measurement is limited to ±3%. The sensitivity of the temperature to the maximum uncertainty in flow measurement is 3.5% which makes the present level of accuracy acceptable.

3. Mathematical model

In the present work, the experimental results are compared to a mathematical model that takes the effect of flow maldistribution and backmixing into account by introducing an apparent fluid conduction in the flow direction represented by axial dispersion coefficient, λ which is assumed to follow Chester's [14] hyperbolic conduction law in the energy equation.

Thus the governing equations are given by

$$\frac{1}{\alpha^*}\left[\frac{\partial T_f}{\partial \tau}+u\frac{\partial T_f}{\partial x}\right]+\frac{1}{c^{*2}}\left[\frac{\partial^2 T_f}{\partial \tau^2}+2u\frac{\partial^2 T_f}{\partial \tau\,\partial x}+u^2\frac{\partial^2 T_f}{\partial x^2}\right]-\frac{\partial^2 T_f}{\partial x^2} = \frac{hA}{LA_f\lambda}\left[(T_m-T_f)+\frac{\alpha^*}{c^{*2}}\frac{\partial(T_m-T_f)}{\partial \tau}+\frac{u\alpha^*}{c^{*2}}\frac{\partial(T_m-T_f)}{\partial x}\right] \tag{3}$$

$$A_m\rho_m C_m\frac{\partial T_m}{\partial \tau}=A_m k\frac{\partial^2 T_m}{\partial x^2}+\frac{hA}{L}(T_f-T_m) \tag{4}$$

The dispersion parameters λ and c^* and heat transfer coefficient h are non-dimensionalised using the following parameters.

Dispersive Peclet number $= Pe = \dfrac{m_f C_p L}{\lambda A_f}$

Third sound Mach number $= V = \dfrac{u}{c^*}$

Number of transfer units $= \mathrm{NTU} = \dfrac{hA}{m_f C_p}$

With these parameters the governing fluid and the solid (packing material) equations in non-dimensionalised form can be expressed as,

$$\frac{Pe}{\eta}\frac{\partial \theta_f}{\partial z}+Pe\frac{\partial \theta_f}{\partial X}+\frac{V^2}{\eta^2}\frac{\partial^2\theta_f}{\partial z^2}+\frac{2V^2}{\eta}\frac{\partial^2\theta_f}{\partial z\,\partial X}+(V^2-1)\frac{\partial^2\theta_f}{\partial X^2} = Pe\mathrm{NTU}(\theta_m-\theta_f)+\frac{\mathrm{NTU}V^2}{\eta}\frac{\partial(\theta_m-\theta_f)}{\partial z}+\mathrm{NTU}V^2\frac{\partial(\theta_m-\theta_f)}{\partial X} \tag{5}$$

$$\frac{\partial \theta_m}{\partial z}=\frac{1}{P_m}\frac{\partial^2\theta_m}{\partial X^2}+\mathrm{NTU}(\theta_f-\theta_m) \tag{6}$$

It should be noted here that a heterogenous model which accounts for difference in fluid and solid matrix temperature at each axial location is used here. Thus the heat transfer coefficient approach which is usually used for the analysis of thermal regenerators is used here instead of heat transfer resistance approach of homogeneous model.

Danckwert [15] type of boundary condition is extended to take care of the finite propagation velocity of dispersion as

$$T_{in}^{(+)}-T_{in}^{(-)}=\frac{\lambda}{\rho u\, C_p}\frac{\partial T_{in}^{(+)}}{\partial x}-\frac{\alpha^*}{c^{*2}}\left[\frac{\partial T_{in}^{(+)}}{\partial \tau}+u\frac{\partial T_{in}^{(+)}}{\partial x}\right] \tag{7}$$

This can be non-dimensionalised as

$$\theta_f^{(+)}-\frac{(1-V^2)}{Pe}\frac{\partial \theta_f^{(+)}}{\partial X}+\frac{V^2}{Pe\eta}\frac{\partial\theta_f^{(+)}}{\partial z}=\theta_f^{(-)}\quad \text{at } X=0 \tag{8}$$

$$\frac{\partial\theta_f}{\partial X}=0\quad \text{at } X=1 \tag{9}$$

Physically this type of boundary condition indicate that when a fluid with plug flow suddenly enters a dispersive region there is a sudden drop in temperature. This drop is dependent on the level of the dispersion (i.e. on the dispersive Peclet number). In case the approaching fluid is also considered with dispersion, only the level of dispersion will change.

It should be mentioned here that, strictly speaking Danckwerts boundary condition is valid only if there is a plug flow before and after the packed section. However, in the present case the flow before and after packing was fully turbulent giving a very flat velocity profile. Also, these sections were properly insulated to have a similar temperature profiles which can be approximated as plug flow without incurring much error. However, for large deviation from plug flow in the fore and aft section this boundary condition can be modified by incorporating the respective fore and aft Peclet number which will not change the general conclusions of the present work in a significant way.

The matrix conduction at inlet and exit are absent. The non-dimensional form of matrix boundary condition is given by

$$\frac{\partial \theta_{\mathrm{m}}}{\partial X} = 0 \quad \text{at } X = 0 \text{ and } X = 1 \tag{10}$$

Initially, both the fluid and the matrix are at atmospheric temperature. Incorporating this initial condition, the fluid and the matrix equations can be transformed into Laplace domain which can be written in matrix form as

$$\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}X} = \overline{A}\bar{\theta} \tag{11}$$

where, the temperature vector $\bar{\theta}$ is given by

$$\bar{\theta} = \left[\theta_{\mathrm{f}}(s), \frac{\mathrm{d}\theta_{\mathrm{f}}(s)}{\mathrm{d}X}, \theta_{\mathrm{m}}(s), \frac{\mathrm{d}\theta_{\mathrm{m}}(s)}{\mathrm{d}X}\right]^{\mathrm{T}} \tag{12}$$

and $\overline{A}$ is a 4×4 co-efficient matrix formed from the coefficients of the temporal fluid and matrix equation. Eq. (12) can be solved by an identical way as done by Das et al. [11] to yield the solution of the form

$$\bar{\theta} = U\beta(X)D \tag{13}$$

where the diagonal matrix $\beta(X)$ is given by

$$\beta(X) = \mathrm{diag}\{\mathrm{e}^{\beta_1 X}, \quad \mathrm{e}^{\beta_2 X}, \quad \mathrm{e}^{\beta_3 X}, \quad \mathrm{e}^{\beta_4 X}\} \tag{14}$$

The columns of matrix U are the eigenvectors of the matrix $\overline{A}$ and D is a co-efficient vector given by

$$D = [d_1, \quad d_2, \quad d_3, \quad d_4]^{\mathrm{T}} \tag{15}$$

which can be evaluated from the boundary conditions. Putting boundary conditions in Eq. (13) will given a matrix equation for which $\bar{\theta}$ is known. This can be solved by standard matrix solvers (here Gaussian elimination is used) to give D.

The solution which is obtained using the above method is in frequency domain and it is inverted back to the time domain using fast Fourier transform technique suggested by Crump [16].

4. Results and discussion

A wide range of experiments have been carried out for three different sizes of stainless steel (16×16, 10×10, 5×5) wire mesh packed in a circular tube. The porosities for these meshes are 0.766, 0.817 and 0.832 respectively. Because of the limitation of the flow rate, the Reynolds number are obtained in different ranges as shown in Fig. 5. Heat transfer data for the matrices are taken from Kays and London [17].

5. Measurement of dispersion parameters (apparent Peclet number *Pe* and third sound Mach number *V*)

Figs. 3 and 4 show the comparison of computed response using the present model at different Pe and V with the experimental response for two different meshes (16×16, and 5×5). Pe and V of the computed response at which the time constant is closest to the experimental response is taken as the dispersion parameter at that particular Reynolds number. Using the above procedure Pe and V are measured for different Reynolds numbers and mesh sizes. Fig. 5 shows measured Pe and V for three mesh sizes and different ranges of Reynolds number. A generalised correlation is suggested for Pe and V in terms of Re and the bed porosity (p) using a non-linear regression analysis which can be given by,

$$Pe = 10.13 Re^{-0.55} p^{3.93} \tag{16}$$

$$V = 0.45 Re^{0.04} p^{-1.43} \tag{17}$$

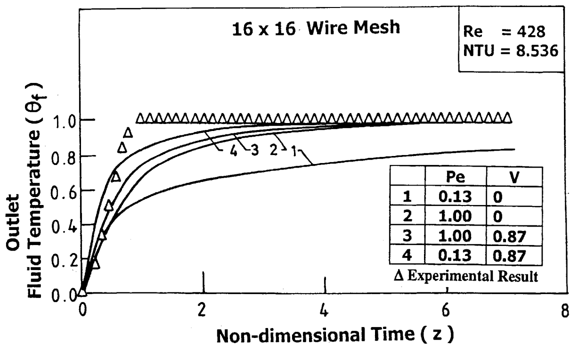

Fig. 3. Comparison of the experimental outlet fluid temperature with the present model at different Pe and V for 16×16 wire mesh ($Re = 428.3$, NTU $= 8.536$).

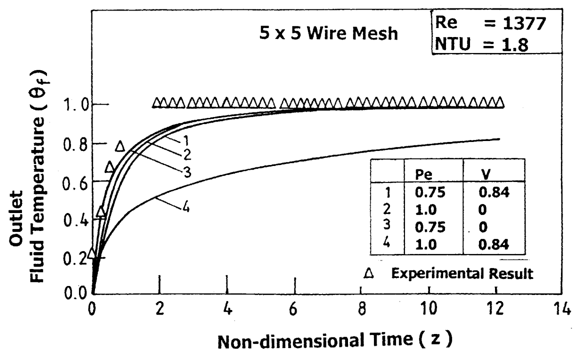

Fig. 4. Comparison of the experimental outlet fluid temperature with the present model at different Pe and V for 5×5 wire mesh ($Re = 1377.33$, NTU $= 1.8$).

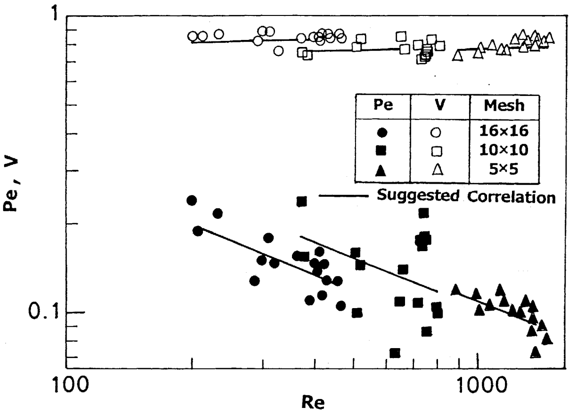

Fig. 5. Predicted values of Pe and V with fitted correlation in terms of Reynolds number and matrix porosity for 16×16, 10×10, 5×5 wire meshes.

for the range of parameters

$200 < Re < 1600$

$0.766 < p < 0.832$

These correlations are applicable only for the meshes used here and the range of Re shown in Fig. 5. The focus of the present study is to indicate the possibility of obtaining such correlations rather then generating a general correlation which is left as a scope of future work.

6. Validation of the present model

By using the proposed correlation, the response of the bed is calculated and compared with an experimental transient response which has not been used for obtaining the correlation. Fig. 6 shows the comparison of the calculated response and the experimental response. The result is convincing to reveal the validity and consistency of the hyperbolic dispersion proposition. To further demonstrate the appropriateness and contribution of the present formulation, in Figs. 7 and 8, comparison of experimental result with the present model, the parabolic dispersion model ($V = 0$) and the plug flow model is made. For 16×16 wire mesh, i.e. higher NTU, the present model gives a better approximation to the reality. At lower NTU (5×5) there is not much difference between the present model, parabolic dispersion model and the plug flow model. This is due to the fact that at lower NTU (5×5 mesh), heat transfer to the wire mesh is less. So the temperature gradient in the fluid along the bed is not large. Consequently, the heat transfer due to dispersion does not play a major role.

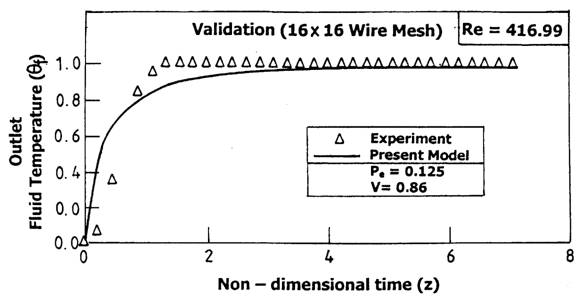

Fig. 6. Validation of the present model with 16×16 wire mesh.

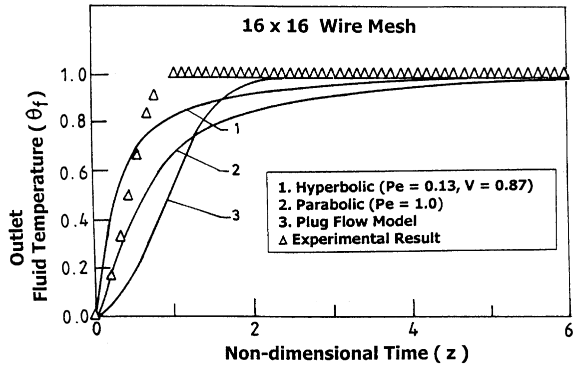

Fig. 7. Comparison of the experimental outlet fluid temperature response with present model, parabolic dispersion model and plug flow model for 16×16 wire mesh.

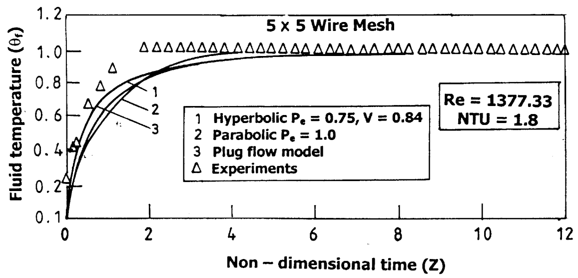

Fig. 8. Comparison of the experimental outlet fluid temperature response with present model, parabolic dispersion model and plug flow model for 5×5 wire mesh.

Fig. 9 compares purely the experimental outlet fluid temperature at same range of Reynolds number for 16×16 and 10×10 wire mesh. For 16×16 mesh, the outlet fluid temperatures rises faster as NTU for this wire mesh is higher and consequently the matrix reaches the fluid temperature earlier.

This feature suggests the importance of the different models for different types of heat transfer equipment. Eventhough in packed bed the flow maldistribution is less severe, at high NTU (i.e., low porosity) the dispersion becomes significant. Hence for equipment in which flow maldistribution is more or compactness is higher, it is essential to include the hyperbolic dispersion model. For devices such as packed bed with moderate to high

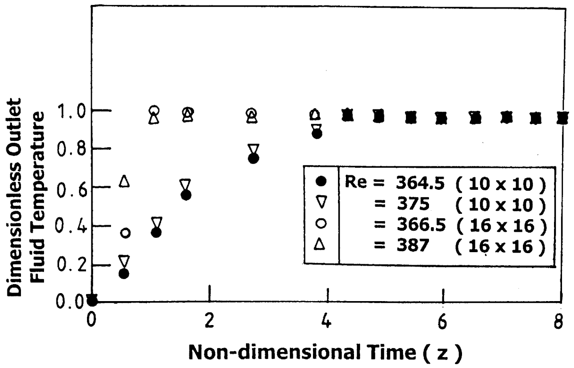

Fig. 9. Comparison of the experimental outlet fluid temperature responses of 16 × 16 and 10 × 10 wire mesh at same range of Reynolds number.

porosity, the usual (parabolic) dispersion model seems to be sufficient while plug flow seems to be poor approximation in all cases involving heat transfer.

7. Matrix temperature distribution

Figs. 10 and 11 show matrix temperature history at two different locations ($X = 0.1875$ and 0.8125) of the bed in the flow direction. Similar to the observation from the fluid temperature response the matrix temperature of the model gives a good approximation of the matrix temperature. Fig. 12 shows the standard deviation of the matrix temperature distribution of the present model with experimental response with respect to time considering temperatures at three locations of the bed ($X = 0.1875$, 0.5 and 0.8125).

Compact heat exchangers such as regenerators require higher NTU. The results show that at higher NTU, the dispersion and its propagation velocity play an important role and the present model matches better with the experimental response than the parabolic dis-

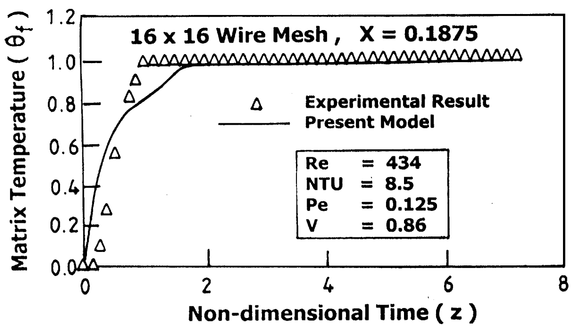

Fig. 10. Comparison of experimental matrix temperature history with the calculated matrix temperature response with respect to time for 16 × 16 wire mesh at $X = 0.1875$.

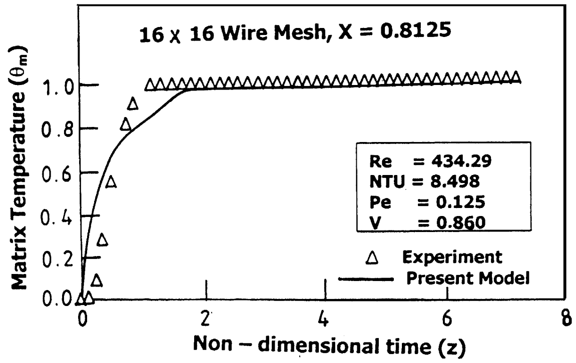

Fig. 11. Comparison of experimental matrix temperature history with the calculated matrix temperature response with respect to time for 16 × 16 wire mesh at $X = 0.8125$.

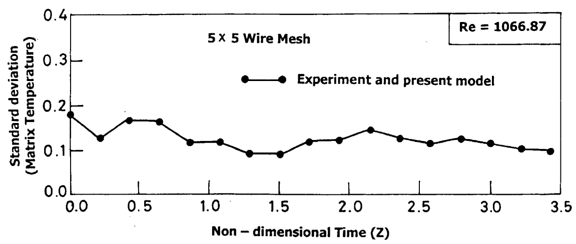

Fig. 12. The standard deviation of the matrix temperature distribution of the present model with experimental response with respect to time.

persion model (and of course the plug flow model). Hence, the present experimental technique can be extended and general experimental correlations for *Pe* and *V* can be obtained for a range of *Re* and other types of packing geometries (such as metallic spheres and finned surfaces) which can be used for simulating a packed bed thermal regenerator and compact heat exchangers.

8. Conclusion

The concept of hyperbolic dispersion is established by theoretical analysis and by conducting experiments on packed bed systems. The hyperbolic dispersion model is validated using the predicted *Pe* and *V* from the proposed correlations. Experimental result reveals that the present model is a good approximation to the transient thermal behaviour of a packed bed energy reservoir especially at higher NTU which makes the present study important as compact heat exchangers which are used only at higher NTU. Matrix temperature distribution calculated are on the whole consistent with experimental result. A standard experimental technique for the measurement of parameters to axial dispersion and its propagation is suggested which can be extended for presenting a detailed data for packed bed regenerators and other compact heat exchanger surfaces.

The dispersion parameters, viz., dispersive Peclet number (*Pe*) and the third sound Mach number (*V*) are predicted for a range of Reynolds numbers for three different sizes of wire meshes and a pair of general correlation are suggested for them.

References

[1] S.G. Taylor, The dispersion of matter in turbulent flow through a pipe, Proceedings of the Royal Society of London 223A (1954) 446–468.

[2] W. Roetzel, B. Spang, Axiales Dispersionsmodell fur Regeneratoren mit endicher Aubreitungsgeschwindigkeit thermischer storungen, Mitteilung aus dem. Institute of thermodynamics, University of Federal Armed Forces, Hamburg, Germany, 1994.

[3] G.A. Adebiyi, D.J. Chenevert, An appraisal of one-dimensional analytical models for the packed bed thermal storage systems utilising sensible heat storage material, Journal of Energy Resources, Technology, Transactions of ASME 118 (1996) 44–49.

[4] T.E.W. Schumann, Heat transfer, liquid flowing through a porous prism, Journal of Franklin Institute 208 (1929) 405–416.

[5] M. Riaz, Transient analysis of packed-bed thermal storage systems, Solar Energy 21 (1978) 123–128.

[6] A. Montakhab, Convective heat transfer in porous media, Journal of Heat Transfer, Transactions of ASME 101 (1979) 507–510.

[7] P.-C. Lu, Single-blow transients in packed bed storage units, with solid-phase conduction by Crump's numerical inversion of Laplace transforms, Journal of Heat Transfer, Transactions of ASME 105 (1983) 493–497.

[8] W. Roetzel, Y. Xuan, Analysis of transient behaviour of multipass shell and tube heat exchangers with the dispersion model, International Journal and Heat and Mass Transfer 35 (1992) 2953–2992.

[9] Y. Xuan, W. Roetzel, Measurement of heat exchangers to arbitrary temperature and step flow variations, AICHE Journal 39 (1993) 413–421.

[10] W. Roetzel, X. Luo, Y. Xuan, Measurement of heat transfer coefficient and axial dispersion coefficient using temperature oscillations, Experimental Thermal and Fluid Science 7 (1993) 345–353.

[11] S.K. Das, B. Spang, W. Roetzel, Dynamic behaviour of plate heat exchangers—experiments and modelling, Journal of Heat Transfer, Transactions of ASME 117 (1995) 859–864.

[12] W. Roetzel, F. Balzereit, Determination of axial dispersion coefficients in plate heat exchangers using residence time measurements, in: Engineering Foundation Conference on Compact Heat Exchangers for the Process Industries, Snowbird, Utah, USA, June 22–27 1997, pp. 635–643.

[13] W. Roetzel, X. Luo, B. Spang, S.K. Das, Propagation of third sound wave in fluid, hypothesis and theoretical foundation, International Journal of Heat and Mass Transfer 41 (1998) 2769–2780.

[14] M. Chester, Second Sound in Solids, Physical Review 131 (1963) 2013–2015.

[15] P.V. Danckwert, Continuous flow systems—distribution of residence times, Chemical Engineering Science, Genie Chimique 2 (1953) 1–13.

[16] K.S. Crump, Numerical inversion of Laplace transforms using a Fourier series approximation, Journal of the Association for Computing Machinery 23 (1976) 89–96.

[17] W.M. Kays, A.L. London, Compact Heat Exchangers, third ed., McGraw-Hill, New York, 1984.